Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 12: Probability I

12.1 Learning Intentions

After this week's lesson you will be able to;

- Use the fundamental principle of counting to show permutations.
- Explain what is meant by factorial.
- Evaluate the number of combinations possible for a set.
- Use a sample space to represent events.
- Calculate the experimental probability (RF).
- Calculate expected frequency (EF).
- Verify if an event is mutually exclusive.
- Multiplication of independent events

12.2 Specification

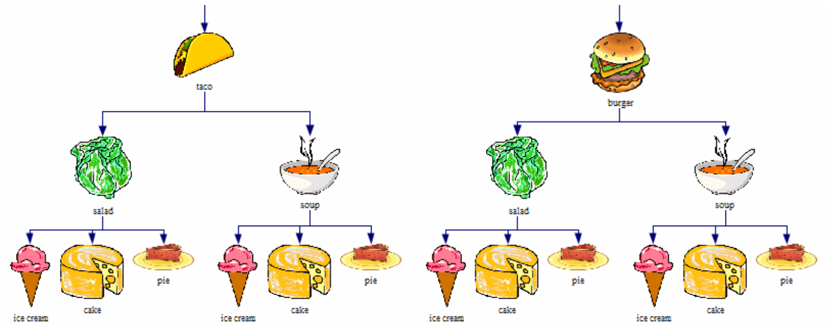
Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	– count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects	– count the number of ways of selecting r objects from n distinct objects – compute binomial coefficients
1.2 Concepts of probability	– use set theory to discuss experiments, outcomes, sample spaces – discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams – calculate expected value and understand that this does not need to be one of the outcomes – recognise the role of expected value in decision making and explore the issue of fair games	– extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae – Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ – Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ – Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$ – solve problems involving sampling, with or without replacement – appreciate that in general $P(A \mid B) \neq P(B \mid A)$ – examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context
1.3 Outcomes of random processes	– find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli trials – calculate the probability that the 1st success occurs on the n^{th} Bernoulli trial where n is specified	– solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required) – calculate the probability that the k^{th} success occurs on the n^{th} Bernoulli trial – use simulations to explore the variability of sample statistics from a known population, to construct sampling distributions and to draw conclusions about the sampling distribution of the mean – solve problems involving reading probabilities from the normal distribution tables

12.3 Chief Examiner's Report

B	8	35·3	54	9	Probability

12.4 Counting

We can use the fundamental principle of counting to show the number of options.



List below how to calculate the number of options for dinner:

12.5 Factorial

This can be used to calculate a certain number multiplied by every natural number between itself and 1 inclusive. For example

7 x 6 x 5 4 x 3 x 2 x1 can be described as 7 Factorial, or simply 7!

On the calculator (Casio) look for this button:

To access this button, you press 7 then shift and then this button.

12.6 Permutations

This is the number of ways arranging a number of objects (n) taking a certain number at a time (r).

To calculate the number of arrangements we use the below button on the calculator. These permuations allow for the same number of items to be selected but in a different order. A fact that becomes more obvious when we look at combinations.

To use this button, we type the number of items first (n) then press shift and this button followed by the number of items we are choosing at each time (r).

COUNTERS (Fill in answer below):

Another aid can be the pigeonhole principle:

12.7 Combinations

Combinations are however, a little bit more restrictive. These do not allow for repeated combinations even if the order they are in are different. We can see this in the example in the video:

We have 4 seats to be allocated for a board meeting. We have 6 possible staff members to choose from, A, B, C, D, E & F. Using our previous knowledge of permutations we can calculate:

Using nPr = 360

However these 360 include arrangements such as ABCD and ACBD. Which although are in a different order, they are still the same 4 people. The use of combinations and the nCr button we can eliminate the repeated groups.

Using nCr = 15

These 15 don't include any repeated patterns and tells us the number of different combinations of staff we could send to the board meeting.

Sample Questions nCr and nPr:

1) The ski club with ten members is to choose three officers captain, co-captain & secretary, how many ways can those offices be filled?

2) To win the small county lottery, one must correctly select 3 numbers from 30 numbers. The order in which the selection is made does not matter. How many different selections are possible?

3) A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?

4) In a race in which six automobiles are entered and there are not ties, in how many ways can the first four finishers come in?

12.8 Language of Probability

Experiment: The thing that is happening, i.e. flipping a coin, rolling a die etc.

Trial: The act of carrying out the experiment.

Outcome: The result of a trial.

Sample Space The set of all possible outcomes to a trial.

12.9 Experimental Probability

This is the real life probability. This is calculated after the experiment has taken place and is a record of the outcomes and their occurrence during the experiment.

This can be accurate reflection of the true probability of an event provided the number of trials is sufficiently large.

$$Relative\ Frequency = \frac{frequency\ of\ event\ in\ the\ trials}{total\ number\ of\ trials}$$

R.F. of all outcomes in an event should sum to 1.00

12.10 Theoretical Probability

This can be referred to as expected frequency, as in what we should get following an event in an ideal world.

To calculate this probability, we do the following:

$$Probability = \frac{number\ of\ elements\ in\ the\ event}{number\ of\ outcomes\ of\ the\ event}$$

For instance, if we look at the events in the video we can see the following expected frequencies:

$$P(Spade) = \frac{13}{52}$$

$$P(King) = \frac{4}{52}$$

$$P(King\ or\ a\ Spade) = \frac{16}{52}$$

Describe the relationship between this **or** probability and the probabilities of the individual events above:

Relationship =

Describe the relationship between this **or** probability and the probabilities of the individual events above:

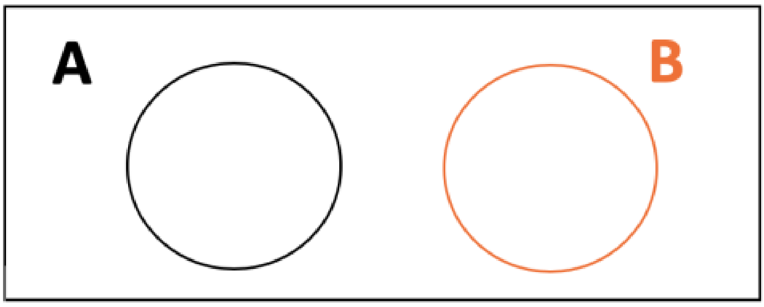
Relationship =

$$P(King\ and\ a\ Spade) = \frac{1}{52}$$

Describe the relationship between this **and** probability and the probabilities of the individual events above:

Relationship =

12.11 Mutually Exclusive



If these events are mutually exclusive, this means that they cannot happen at the same time. In other words, using set notation we can see that there is no intersection of the two events, i.e. the probability of both events occurring together is 0.

$P(A \cap B) = 0$

Therefore if two events are mutually exclusive the following should be true:

$P(A \cup B) = P(A) + P(B)$

If two events are not mutually exclusive then the following will be true:

$P(A \cup B) > P(A) + P(B)$

The union of the two events will be greater by a value that is equal to the value of the intersection of the two events.

12.12 Conditional Probability

This is where we are given an initial condition for our event

For example:

There are 20 vehicles in a garage, 12 Cars and 8 SUV's
9 of these Vehicles are Volkswagen made

What's the probability of choosing a car at random given that it is VW made?

Firstly, to calculate this we need the formula for such a situation:

$$P(A \mid B) = \frac{\#(A \cap B)}{\#(B)} = \frac{P(A \cap B)}{P(B)}$$

Draw the Venn Diagram from the video below: **(PAUSE VIDEO)**

12.13 Independent Events

Two events are independent if the occurrence of one event does not influence the other.

In the event of flipping a coin twice. A heads on the first flip has no impact on the outcome of the second flip. Therefore, the probability of getting a heads on the first flip is independent to getting a tails on the second flip.

So in mathematical terms, if two events are independent then the following should hold true:

$$P(A \cap B) = P(A)P(B)$$

On the opposite side of the scale we could consider picking a card out from a deck at random. If we don't replace the card, we take out on the first instance then this will impact the probability of the outcome for the second trial.

12.14 Recap of the Learning Intentions

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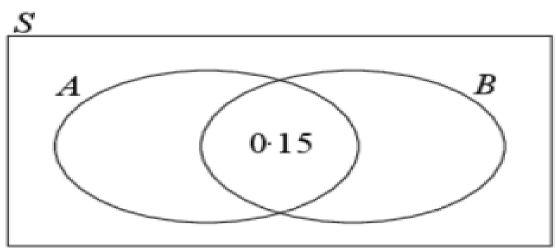
12.15 Homework Task

Two events A and B are such that P(Á)=0.2, P(A∩B)=0.15 and P(Á∩B)=0.6.

a. Complete this Venn diagram



b. Find the conditional that neither A nor B happens.

c. Find the conditional probability P(A|B).

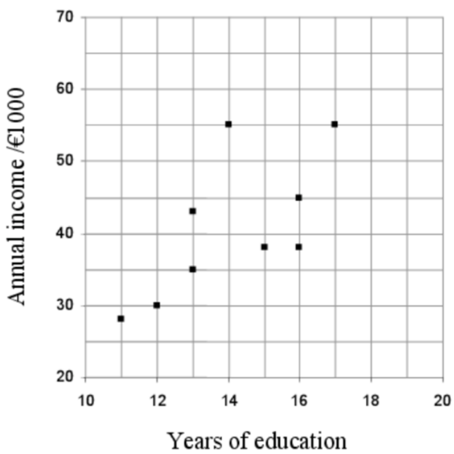
d. State whether A and B are independent events and justify your answer.

12.16 Solutions to 11.12

Question: **75 mrks**

An ecnomics studnet wants to find out whether the lenght of time people spend in education affects the income they earn. The student out a small study. Twelve adults are asked to state their annual inocme and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

Years of education	Income /€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58



i. The last three rows of data have not been include on the scatter plot. Insert them now.

ii. Calculate the correlation coefficient.

Answer: 0.623

iii. What can you conclude form the scatter plot and the correlation coefficient?

Moderate positive correlation as the data loosely follows a linear upwards trend. This matches up well with the r-value as this also implies a slight positive correlation.

iv. Add the line of best fit to the completed scatter plot above.

v. use the line of best fit to estimate the annual income of somebody who has spent 14 years in education.

Answer: €41,000

vi. By taking suitable readings from your diagram, or otherwise, calculate the slope of the line of best fit.

Use a point we have just figured out (14, 41) and another easy one (17, 50)

$$m = \frac{50-41}{17-14} = \frac{9}{3} = 3$$

vii. Explain how to interpret this slope in this context.

As the slope is 3 or $\frac{3}{1}$ this means for every increase of 1 unit in the x-axis (years in education) I get an increase of 3 units in the y-axis (Annual income). So for every year in education I gain an additional €3000 in annual salary